



Relation between proton decay and PMNS phase in the minimal SUSY $SO(10)$ GUT



Takeshi Fukuyama^a, Koji Ichikawa^{b,*}, Yukihiro Mimura^c

^a Research Center for Nuclear Physics (RCNP), Osaka University, Ibaraki, Osaka, 567-0047, Japan

^b University of Tokyo, Kavli Institute for the Physics and Mathematics of the Universe (KIPMU), 5-1-5 Kashiwa-no-Ha, Kashiwa Shi, Chiba 277-8568, Japan

^c Department of Physics, National Taiwan University, Taipei, 10617, Taiwan

ARTICLE INFO

Article history:

Received 30 September 2016

Accepted 8 November 2016

Available online 14 November 2016

Editor: M. Cvetič

ABSTRACT

Proton decay is one of the most important predictions of the grand unified theory (GUT). In the supersymmetric (SUSY) GUT, proton decays via the dimension-five operators need to be suppressed. In the $SO(10)$ model where $\mathbf{10} + \mathbf{\overline{126}}$ Higgs fields couple to fermions, neutrino oscillation parameters including the CP-violating Pontecorvo–Maki–Nakagawa–Sakata (PMNS) phase can be related to the Yukawa couplings to generate the dimension-five operators in the unified framework. We show how the suppressed proton decay depends on the PMNS phase, and stress the importance of the precise measurements of the PMNS phase as well as the neutrino 23-mixing angle. These become especially important if the SUSY particles are found around less than a few TeV at LHC and proton decays are observed at Hyper-Kamiokande and DUNE experiments in the near future.

© 2016 The Author(s). Published by Elsevier B.V. This is an open access article under the CC BY license (<http://creativecommons.org/licenses/by/4.0/>). Funded by SCOAP³.

1. Introduction

The measurement of the parameters in the neutrino oscillations is one of the recent major progress in particle physics. Neutrinos must be very light compared to the other fermions in the standard model (SM). The lightness of neutrinos can be explained by the seesaw mechanism [1,2], which can open a window to the ultra high scale physics. In that sense, the precise measurements of the neutrino mixing angles and the PMNS phase [3–6] are important not only for the scientific interests to archive the accurate values, but also to probe the physics beyond the SM. In fact, their precise measurements are very important to determine the detail structure of the neutrino mass matrix.

The quantum numbers of quarks and leptons are disparate each other, and it has been discussed whether quarks and leptons can be described in a unified framework. In the $SO(10)$ GUT, the entire fermion species (including a right-handed neutrino) can be unified in one spinor multiplet in each generation, and the description of fermion masses and mixings can be developed in a unified framework. There are several ways to construct a model in the $SO(10)$ GUT. Among them, the model with $\mathbf{\overline{126}}$ Higgs representation has an attractive feature that the fermion coupling to $\mathbf{\overline{126}}$ can generate both left- and right-handed Majorana neutrino masses as well as a part of the Dirac masses of the fermions. In that model, the smallness of quark mixing angles and two large neutrino mixings can be naturally explained if the $\mathbf{10}$ -dimensional Higgs coupling generates the third generation Yukawa couplings dominantly [7].

Proton decay is one of the most important predictions of GUTs. SUSY can solve the hierarchy problem between the weak scale and the GUT scale. The SUSY GUTs, however, suffer from constraints of dimension-five proton decay operators [8], which can be generated by the Yukawa interaction via color triplet Higgs exchange: More than 10 years ago, one of the authors and his collaborators discussed how the proton decay amplitudes can be suppressed by the specific Yukawa structure, which can be connected to the parameters in the neutrino oscillations, and claimed that the neutrino 13-mixing angle θ_{13} cannot be zero and $\theta_{13} \sim 0.1$ under the condition [9]. In these arguments, it was assumed that light neutrino mass matrix is dominated by the type II contribution from the left-handed Majorana neutrino mass. In Ref. [10], we describe the sum rule of the fermion mass matrices in the form that the coupling matrices are given by the parameters

* Corresponding author.

E-mail address: koji.ichikawa@ipmu.jp (K. Ichikawa).

in the lepton sector. Using the algebraic expression, we argue that the similar Yukawa structure can be applied even if the type I seesaw contribution is incorporated to the neutrino mass matrix in the case where the vacuum expectation value (VEV) of $\mathbf{126}$, v_R , is large.

In this paper, we describe the Yukawa structure which is suitable to suppress the proton decay amplitude in the $SO(10)$ model, and study how the Yukawa structure depends on the neutrino oscillation parameters. In order to explain the proton decay suppression, we adopt the minimal $SO(10)$ model, in which the $\mathbf{10} + \mathbf{126}$ Higgs fields couple to fermions [11–18]. We use the χ^2 fits in the minimal $SO(10)$ model, obtained in Ref. [10] to show the consequence numerically. In the case where $v_R \gtrsim 10^{16}$ GeV, the suppressed proton decay width depends on the parameters, and we show the numerical calculation in terms of the neutrino 23-mixing angle and the PMNS phase. The accurate measurements of the oscillation parameters are important if the dimension-five proton decays are suppressed but it will be observed in the near future.

2. The importance of the PMNS phase to the structure of the neutrino mass matrix

CP violation of the neutrino oscillations is important to learn the nature of the fundamental symmetry. Not only because of the fundamental curiosity, but also to understand the structure of the neutrino mass matrix the precise measurement of the PMNS phase is necessary.

In the quark sector, the mixing angles are all small, and the hierarchical structure of the quark mass matrices can be simply obtained. The possible exception is the (1, 1) element of the mass matrix. When the mass matrix is reconstructed from various observations, there may be a cancelation in the (1, 1) element. It is well-known that there is an empirical relation between the quark mass ratio and the Cabibbo angle

$$\sin \theta_C \simeq \sqrt{\frac{m_d}{m_s}}, \quad (1)$$

which may be related to the smallness of the (1, 1) element (compared to down quark mass) of the down-type quark mass matrix in the basis where up-type quark mass matrix is diagonal. In the lepton sector, on the other hands, the neutrino mixing angles are large and the hierarchical structure is not obvious yet. As in the quark sector, there may be a cancelation when the neutrino mass matrix is reconstructed from observations.

The neutrino mass matrix (in the basis where the charged lepton mass matrix is diagonal) is expressed as

$$\mathcal{M}_\nu = U \text{diag.}(m_1, m_2, m_3) U^T, \quad (2)$$

where U is the unitary mixing matrix determined by three neutrino mixing angles $\theta_{12}, \theta_{23}, \theta_{13}$ and a phase δ_{PMNS} . Using a convention by Particle Data Group [19], we obtain the (1, 1) and (1, 2) elements as

$$(\mathcal{M}_\nu)_{11} = (m_1 \cos^2 \theta_{12} + m_2 \sin^2 \theta_{12}) \cos^2 \theta_{13} + e^{-2i\delta_{\text{PMNS}}} m_3 \sin^2 \theta_{13}, \quad (3)$$

$$(\mathcal{M}_\nu)_{12} = \cos \theta_{13} \left[(m_2 - m_1) \cos \theta_{12} \sin \theta_{12} \cos \theta_{23} + e^{-i\delta_{\text{PMNS}}} \sin \theta_{13} \sin \theta_{23} (m_3 - e^{2i\delta_{\text{PMNS}}} (m_1 \cos^2 \theta_{12} + m_2 \sin^2 \theta_{12})) \right]. \quad (4)$$

In this expression, mass eigenvalues m_1, m_2, m_3 are complex, and one of three can be made to be real. Because $\sin \theta_{13}$ and $\sqrt{\Delta m_{\text{sol}}^2 / \Delta m_{\text{atm}}^2}$ are the same order, it is possible that both (1, 1) and (1, 2) elements are canceled¹ in the normal hierarchy (NH) case, $\sqrt{\Delta m_{\text{sol}}^2 / \Delta m_{\text{atm}}^2} \sim m_2 / m_3$. We remark that the PMNS phase has to be chosen if those elements vanishes contrary to the quark sector, and thus the accurate measurement of the PMNS phase is quite important to determine the structure of the neutrino mass matrix.

Let us suppose that both (1, 1) elements and (1, 2) elements are completely zero [20,21]

$$(\mathcal{M}_\nu)_{11} = (\mathcal{M}_\nu)_{12} = 0, \quad (5)$$

to see what condition is needed to suppress those elements. There are four equations for real degree of freedom. We obtain a relation among the phase δ_{PMNS} and mixing angles $\theta_{12}, \theta_{23}, \theta_{13}$ and the ratio of the mass squared differences $\Delta m_{12}^2 / \Delta m_{23}^2$, as well as $|m_1|$ and two Majorana phases of the mass eigenvalues. For the point of view that the constraint can be verified from the neutrino oscillations, the relation among the phase, mixing angles and masses is the most important. We obtain [22,23]

$$\cos \delta_{\text{PMNS}} = \frac{\frac{\Delta m_{\text{sol}}^2}{\Delta m_{\text{atm}}^2} \cos 2\theta_{13} \sin^2 2\theta_{12} - 4 \sin^2 \theta_{13} \left(\frac{\Delta m_{\text{sol}}^2}{\Delta m_{\text{atm}}^2} \cos^4 \theta_{12} + \cos 2\theta_{12} \right) \tan^2 \theta_{23}}{4 \sin^3 \theta_{13} \left(1 + \frac{\Delta m_{\text{sol}}^2}{\Delta m_{\text{atm}}^2} \cos^2 \theta_{12} \right) \sin 2\theta_{12} \tan \theta_{23}}. \quad (6)$$

The atmospheric neutrino oscillations constrains $\sin 2\theta_{23}$ (up to matter effects), and thus $\tan \theta_{23}$ still has a large ambiguity. We plot the relation between δ_{PMNS} and θ_{23} in Fig. 1 using the experimental measurements for the other parameters. The experimental global fit results are overlapped in the figure. One can see that the current experimental results are consistent with the conditions of the suppressed (1, 1) and (1, 2) elements.

¹ We note that we here claim that the cancelation can happen among the mixing angles and the mass ratio when the mass matrix is reconstructed from the observations. It does not call a fine-tuning issue since the mixing angles and mass eigenstates are obtained from the mass matrix, and the structure of the matrix is more essential. There may be a fundamental symmetrical reason that some of the elements are smaller than the naive size of them, but in this paper, we take a stance only to discuss what can be expected if the elements are suppressed.

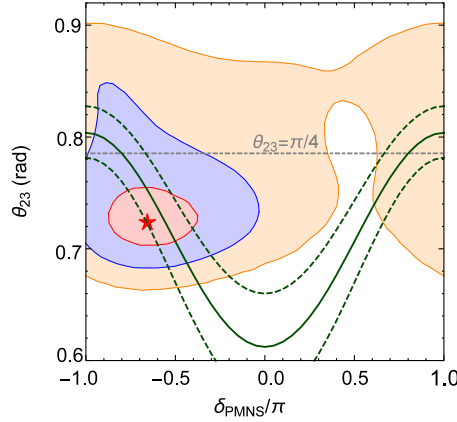


Fig. 1. The plot from the relation in Eq. (6). Along the green line, the (1, 1) and (1, 2) elements can become zero in the neutrino mass matrix in the basis where the charged-lepton mass matrix is diagonal. The dotted lines are drawn using the 3σ range of the mixing parameters. We also overlap the current 1σ (red), 2σ (blue), 3σ (orange) region of the global analysis in Ref. [6]. The star symbol shows the current best fit of the global analysis. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

About a decade ago, one of the authors (Y.M.) and his collaborators argued that the (1, 1) and (1, 2) elements should be small and the 13-mixing angle is predicted to be a non-zero value and about 0.1 if the proton decay amplitudes are suppressed in a renormalizable $SO(10)$ model [9]. Now, the 13-mixing angle is accurately measured, and the key ingredient of the suppression shifts to the accurate values of the PMNS phase and 23-mixing angle. While the PMNS phase lie around $\delta_{\text{PMNS}}/\pi \sim -0.5$ in the global analysis, the 23-mixing angle still have ambiguity depending on which experiments have a major weight. In fact, the global analysis in Refs. [5,6] shows that the best fit lies at $\theta_{23} < \pi/4$, however, it seems that the current best fit without the short-baseline reactor experiments lies at $\theta_{23} > \pi/4$. If the 13-mixing angle is 1σ smaller than the current center value, the best fit would shift to $\theta_{23} > \pi/4$ even in the global analysis. More experimental data are needed.

3. The proton decay suppression in the minimal $SO(10)$ model

In the $SO(10)$ models, the structure of the neutrino mass matrix can be related to the Yukawa couplings of the charged fermions. Therefore, proton decay amplitudes can be related to the neutrino mixing parameters. In this section, we explain how the smallness of the (1, 1) and (1, 2) elements discussed in the previous section is connected to the proton decay suppression using the minimal $SO(10)$ model.

The minimal $SO(10)$ model [11] is defined as that in which the Yukawa interaction is minimal, namely, only $H : \mathbf{10}$ and $\bar{\Delta} : \overline{\mathbf{126}}$ Higgs representations couple to the fermions $\psi : \mathbf{16}$ by renormalizable interaction:

$$W_Y = \frac{1}{2} \mathbf{h}_{ij} \psi_i \psi_j H + \frac{1}{2} \mathbf{f}_{ij} \psi_i \psi_j \bar{\Delta}. \quad (7)$$

Due to the $SO(10)$ algebra, the coupling matrices are symmetric, $\mathbf{h}_{ij} = \mathbf{h}_{ji}$ and $\mathbf{f}_{ij} = \mathbf{f}_{ji}$. The minimality assumption is often addressed even to the Higgs sector to break the $SO(10)$ symmetry, but we impose the minimality just on the Yukawa interactions in this paper.

The Yukawa coupling (after $SO(10)$ symmetry is broken down to SM) is given by linear combinations of the \mathbf{h} and \mathbf{f} matrices, and the fermion mass matrices are obtained as

$$\begin{aligned} M_u &= (h + r_2 f) v_u, \\ M_d &= r(h + f) v_u, \\ M_e &= r(h - 3f) v_u, \\ M_\nu^D &= (h - 3r_2 f) v_u, \end{aligned} \quad (8)$$

where r and r_2 depend on the Higgs mixing (doublet Higgs mixing in $\mathbf{10}$ and $\overline{\mathbf{126}}$), and h and f are original Yukawa matrices \mathbf{h} and \mathbf{f} multiplied by Higgs mixings,

$$h = V_{11} \mathbf{h}, \quad f = \frac{U_{12}}{\sqrt{3}r_1} \mathbf{f}, \quad r = r_1 \frac{v_d}{v_u}, \quad r_1 = \frac{U_{11}}{V_{11}}, \quad r_2 = r_1 \frac{V_{13}}{U_{12}}, \quad (9)$$

where the unitary matrices U and V are the diagonalizing matrices of the doublet Higgs matrix M_{doublet} : $UM_{\text{doublet}}V^T$ is diagonal, and v_u and v_d are the VEVs of up- and down-type Higgs fields. The detail of the convention is found in Ref. [9]. We note that $r \sim m_b/m_t$ if f_{33} is small, and $\tan \beta = v_u/v_d$ is roughly related to r_1 ,

$$\tan \beta \sim r_1 \frac{m_t}{m_b}. \quad (10)$$

The right-handed Majorana neutrino mass matrix is obtained as

$$M_R = \sqrt{2} \mathbf{f} v_R, \quad (11)$$

where v_R is a VEV of $\overline{\mathbf{126}}$. Practically, we denote

$$M_R = c_R v_R f. \quad (12)$$

The seesaw neutrino mass matrix can be written as [1,2]

$$\mathcal{M}_\nu = M_L - M_\nu^D M_R^{-1} (M_\nu^D)^T, \quad (13)$$

where M_L is the left-handed neutrino Majorana mass which comes from $SU(2)_L$ triplet coupling [24], $\ell\ell\Delta_L$. In the $SO(10)$ model, the $\overline{\mathbf{126}}$ Higgs also includes the $SU(2)_L$ triplet Δ_L and the Yukawa coupling generates both M_L and M_R . Therefore, M_L is also proportional to the coupling matrix f and we denote

$$M_L = c_L v_L f. \quad (14)$$

In the $SO(10)$ model, if there is **210** or **54** Higgs representation, the VEV of Δ_L , $\langle\Delta_L\rangle = v_L$, can be obtained as $v_{\text{weak}}^2/M_\Delta$, where M_Δ is the mass of the $SU(2)_L$ triplet.

The dimension-five operator is given in the form of superpotential as

$$-W_5 = \frac{1}{2} C_L^{ijkl} q_k q_l q_i \ell_j + C_R^{ijkl} e_k^c u_l^c u_i^c d_j^c. \quad (15)$$

In the minimal $SO(10)$ model, the dimension-five operators generated by the exchange of the colored triplets, $(\mathbf{3}, \mathbf{1}, -1/3) + (\bar{\mathbf{3}}, \mathbf{1}, 1/3)$, can be written as

$$C_L^{ijkl} = (M_T^{-1})_{11} \mathbf{h}_{ij} \mathbf{h}_{kl} + (M_T^{-1})_{12} \mathbf{f}_{ij} \mathbf{h}_{kl} + (M_T^{-1})_{31} \mathbf{h}_{ij} \mathbf{f}_{kl} + (M_T^{-1})_{32} \mathbf{f}_{ij} \mathbf{f}_{kl} \quad (16)$$

$$= \sum_a \frac{1}{M_{T_a}} (X_{a1} \mathbf{h} + X_{a2} \mathbf{f})_{ij} (Y_{a1} \mathbf{h} + Y_{a3} \mathbf{f})_{kl}, \quad (17)$$

$$C_R^{ijkl} = (M_T^{-1})_{11} \mathbf{h}_{ij} \mathbf{h}_{kl} - (M_T^{-1})_{12} \mathbf{f}_{ij} \mathbf{h}_{kl} \quad (18)$$

$$- ((M_T^{-1})_{31} - \sqrt{2}(M_T^{-1})_{41}) \mathbf{h}_{ij} \mathbf{f}_{kl} + ((M_T^{-1})_{32} - \sqrt{2}(M_T^{-1})_{42}) \mathbf{f}_{ij} \mathbf{f}_{kl} \\ = \sum_a \frac{1}{M_{T_a}} (X_{a1} \mathbf{h} - X_{a2} \mathbf{f})_{ij} (Y_{a1} \mathbf{h} - (Y_{a3} - \sqrt{2}Y_{a4}) \mathbf{f})_{kl}, \quad (19)$$

where X and Y are the diagonalization unitary matrices of the colored triplet mass matrix M_T :

$$X M_T Y^T = \text{diag.}(M_{T_1}, M_{T_2}, \dots, M_{T_5}). \quad (20)$$

We denote that M_{T_1} is the lightest triplet Higgs mass. See Ref. [9] for more details.

The dimension-five proton decay amplitude depends on (1) the masses of SUSY particles, (2) the triplet Higgs mass, and (3) the structure of the Yukawa couplings, for the model parameters. Among them, we here discuss the Yukawa structure to suppress the amplitudes. People often consider the contribution only from the left-handed operator C_L , since the contribution from the right-handed operator C_R is smaller than C_L . However, for the current experimental bounds, there must be a cancelation among the left-handed contributions, and the contribution from C_R itself can exceeds the bounds. In fact, it turns out that the minimal $SU(5)$ model conflicts with the bounds due to the contribution from C_R [25,26]. In $SO(10)$ model, on the other hands, there are additional freedoms to cancel the amplitudes in the triplet Higgs mixings, and the amplitudes can be canceled, which is performed in Ref. [27]. However, the signs of the $\mathbf{f}_{ij} \mathbf{h}_{kl}$ coefficients are opposite between C_L and C_R due to the algebraic reason that the triplet components in H and $\bar{\Delta}$ have opposite “D-parities”. This fact makes the simultaneous cancelation of both C_L and C_R unnatural [9]. More concretely, for the left-handed operator, the triplet Higgs mixing in Eq. (16) can be chosen to obtain $C_L^{ijkl} \sim (Y_u)_{ij} (Y_u + bf)_{kl} / M_{T_1}$ very roughly so that the amplitudes from C_L^{112l} and C_L^{121l} are suppressed. Note that the ij part of the operator in the minimal $SU(5)$ model (without a Higgs mixing) is given as Y_d (or Y_e). Even if the $i = j = 1$ contribution is the size of the down quark Yukawa coupling, it can exceed the current experimental bound. Then, the right-handed operator given in Eq. (18) can be roughly written as

$$C_R^{ijkl} \sim (Y_u + af)_{ij} (Y_u + cf)_{kl} / M_{T_1}, \quad (21)$$

and $a \simeq 2X_{12}$ cannot be zero as long as the charged fermion masses and mixing are fit by the h and f matrices. The right-handed operator generates the four-fermi proton decay operator by Higgsino-dressing, $u_R d_{RS} l \nu_\tau$ and $u_{RS} d_L \nu_\tau$, and thus, the dominant contribution can be obtained as $C_R^{1133} y_t y_\tau V_{ts} + C_R^{1233} y_t y_\tau V_{td}$. As a consequence, if the elements f_{11} and f_{12} are small (in the basis where the quark mass matrices are diagonal), both the left- and right-handed contributions can be suppressed simultaneously.

We have learned that the proton decay amplitude can be suppressed if both f_{11} and f_{12} are small for the given SUSY spectrum and triplet Higgs mass. The next issue is whether the smallness of the elements are realized both in the f matrix and neutrino mass matrix. The simplest realization is the case where the $SU(2)_L$ triplet contribution dominates the neutrino mass matrix: $\mathcal{M}_\nu = M_L$, and the neutrino mass matrix is proportional to f . This can happen if the right-handed neutrino decouples and the type I contribution is negligible. The fermion masses and mixings can be fit in the model that the fermions couples to $\mathbf{10} + \overline{\mathbf{126}} + \mathbf{120}$ Higgs representations and the predictions can be obtained from the proton decay suppression [9,23]. In the minimal model, on the other hand, the triplet dominant neutrino mass does not provide a good fit, and the type I contribution is necessary [17,18]. In general, if the type I term contributes, the f_{11} and f_{12} components are not correlated to the elements in the neutrino mass matrix. However, we can show that the coupling f can

be written as a linear combination of \mathcal{M}_ν , M_e and $\mathcal{M}_\nu M_e^{-1} \mathcal{M}_\nu$ algebraically, and the smallness of those elements can be correlated in the case where ν_R is large ($c_R \nu_R \gtrsim 10^{16}$ GeV) [10]. In the limit of $\nu_R \rightarrow \infty$, we obtain

$$c_L \nu_L f = \frac{1}{2} M_e^{1/2} (K + \sqrt{K^2}) M_e^{1/2}, \quad (22)$$

where

$$K = M_e^{-1/2} \mathcal{M}_\nu M_e^{-1/2}. \quad (23)$$

The square root matrix \sqrt{A} is defined as $(\sqrt{A})^2 = A$, and there are $2^3 = 8$ -fold branches in a 3×3 matrix. As one can see, in the case $\sqrt{K^2} = K$, it corresponds to the triplet dominant case. The detail algebraic description is given in Ref. [10]. On the other hand, for the fit for $c_R \nu_R \sim 10^{13}$ GeV, which can give a best fit actually, f_{11} and f_{12} are not small (irrespective of the elements of the neutrino mass matrix), and the proton decay amplitudes are not suppressed.

Here, we show concrete examples of the f matrix (in GeV unit) obtained in Ref. [10] in the form of $rf\nu_u$ to compare with $M_d = Y_d \nu_d = r\nu_u(h + f)$ in the basis where the down-type quark mass matrix is real (positive) diagonal:

$$1. \ c_R \nu_R = 8.86 \times 10^{16} \text{ GeV}$$

$$rf\nu_u = \begin{pmatrix} 0.000127 - 0.000015i & 0.000250 + 0.000026i & 0.00153 + 0.00565i \\ 0.000250 + 0.000026i & 0.00782 - 0.00141i & 0.0188 - 0.0200i \\ 0.00153 + 0.00565i & 0.0188 - 0.0200i & -0.299 - 0.063i \end{pmatrix}, \quad (24)$$

$$2. \ c_R \nu_R = 1.19 \times 10^{13} \text{ GeV (the best fit)}$$

$$rf\nu_u = \begin{pmatrix} 0.00167 - 0.00007i & -0.000678 + 0.000152i & -0.00163 - 0.0150i \\ -0.000678 + 0.000152i & 0.0312 - 0.00120i & 0.0437 - 0.0567i \\ -0.00163 - 0.0150i & 0.0437 - 0.0567i & -0.0095 - 0.195i \end{pmatrix}. \quad (25)$$

Those two f_{11} components are different by 1 digit. Roughly speaking, the lifetime are different by 2 digit, and the best fit solution does not satisfy the current experimental bound for the squarks mass to be around 2 TeV. In the solution for $c_R \nu_R \gtrsim 10^{16}$ GeV, not only the elements are small, but also the f_{11} and f_{12} elements are correlated to the (1, 1) and (1, 2) elements of the neutrino mass matrix and thus they depends on θ_{23} and δ_{PMNS} as seen in the previous section. As a consequence, we find that the suppressed proton lifetime depends on the neutrino oscillation parameters.

4. Numerical study of the neutrino oscillation parameter dependence of the proton decay

The (1, 1) and (1, 2) elements of the neutrino mass matrix are suppressed along the curve shown in Fig. 1. As explained in the previous section, the smallness of the f_{11} and f_{12} is correlated to the (1, 1) and (1, 2) elements of the neutrino mass matrix for the case of $\nu_R \gtrsim 10^{16}$ GeV. Therefore, we expect that the proton decay can be suppressed along the curve qualitatively. Numerically, even if $(\mathcal{M}_\nu)_{11} = (\mathcal{M}_\nu)_{12}$, the f_{11} and f_{12} elements are not exactly zero due to the type I contribution. Besides, the smallness of f_{11} and f_{12} to suppress the contribution from C_R is needed in the basis where the quark mass matrices are diagonal. Thus, the suppression of the proton decay happens at a bit different from the curve shown in Fig. 1, and $\delta_{\text{PMNS}} - \theta_{23}$ dependence depends on the detail fits of the charged fermion masses and mixings. In this section, we show the numerical calculation of the proton lifetime in order to confirm that the qualitative expectation is realized using the explicit fits given in Ref. [10].

In order to show the numerical calculation of the partial lifetime $p \rightarrow K \bar{\nu}$, we will setup in the following ways. As given in Eqs. (17), (19), the dimension-five operators $C_{L,R}^{ijkl}$ are given by the linear combinations of $h_{ij}h_{kl}$, $f_{ij}h_{kl}$, $h_{ij}f_{kl}$ and $f_{ij}f_{kl}$. We will choose the three coefficients in C_L to cancel the left-handed contribution to the wino-dressed four-fermi nucleon decay amplitudes, $A_L(p \rightarrow K \bar{\nu}_\tau)$, $A_L(n \rightarrow \pi \bar{\nu}_\tau)$, and $A_L(n \rightarrow K \bar{\nu}_\tau)$. The two coefficients in C_R are chosen to cancel the Higgsino-dressed four-fermi decay amplitudes $A_R(n \rightarrow \pi \bar{\nu}_\tau)$, and $A_R(n \rightarrow K \bar{\nu}_\tau)$. As we stressed, the coefficients of $f_{ij}h_{kl}$ have opposite signs in C_L and C_R . Therefore, the coefficients are all fixed by the above conditions up to the overall factor, and the dominant contribution to $p \rightarrow K \nu$ is the Higgsino-dressed four-four fermi operator $(ud)_R(s\nu_\tau)_L$ and $(us)_R(d\nu_\tau)_L$, as explained in the previous section. We note that this method does not provide the minimization of the partial decay width. The partial amplitudes are not necessarily canceled exactly to minimize the decay width. For example, there are additional freedom to cancel the amplitudes between the wino-dressed four-fermi operator $(ud)_L(s\nu_\tau)_L$ and the Higgsino-dressed operator $(ud)_R(s\nu_\tau)_L$, by choosing a phase in the Yukawa coupling, the wino and Higgsino masses. Our purpose to use this method is to show that the qualitative picture given in the previous section appears in the numerical calculation.

For the triplet Higgs mass, we suppose $M_{T_1} \simeq 2 \times 10^{16}$ GeV. Because the triplet Higgs mixings are multiplied, $(M_T^{-1})_{11} \simeq X_{11}Y_{11}/M_{T_1}$ depends on the detail of the triplet Higgs mass matrix. As shown in section 2, the freedom of the bottom Yukawa coupling (namely, the freedom of $\tan\beta$ in MSSM) depends on $r_1 = U_{11}/V_{11}$. Naively, the $\tan\beta$ freedom comes from the couplings $H\Delta\Phi$ and $H\bar{\Delta}\Phi$, where Φ is a **210** Higgs representation, and Y_{11} can relates to the $\tan\beta$ freedom. Therefore, to show the numerical results, we choose $(M_T^{-1})_{11} = m_b \tan\beta / m_t / (2 \times 10^{16}) \text{ GeV}^{-1}$. In the fit in Ref. [10], we use $\tan\beta = 10$. The Higgsino-dressing amplitude of $p \rightarrow K \nu_\tau$ depends on τ Yukawa coupling, and thus the decay amplitude is roughly proportional to $\tan^2\beta$ and the partial lifetime is proportional to $1/\tan^4\beta$ under these setups.

The squark masses and Higgsino mass μ is also important to calculate the proton decay amplitudes. We choose $m_{\tilde{q}} = \mu = 2 \text{ TeV}$. We note that the amplitude is roughly proportional to $\mu/m_{\tilde{q}}^2$. For the parameters of hadron matrix elements, we use numerical values given in Ref. [25].

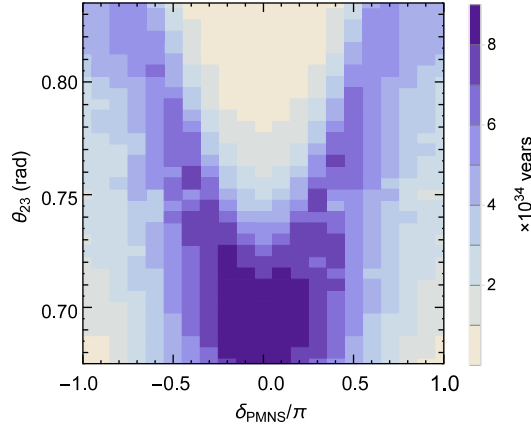


Fig. 2. We show the numerical result of the partial lifetime of the $p \rightarrow K\bar{\nu}$ decay. The detail setup is given in the text. As expected, the lifetime can be larger near the curve given in Fig. 1. The current experimental bound is $\tau(p \rightarrow K\bar{\nu})^{\text{EXP}} > 0.59 \times 10^{34}$ years [28].

In Fig. 2, we show the plot of $\tau(p \rightarrow K\bar{\nu})$ in the $\delta_{\text{PMNS}}-\theta_{23}$ plane. As expected, the partial proton lifetime is larger near the curve given in Fig. 1. Near the curve, the lifetime is about 10 times bigger than the current experimental bound $\tau(p \rightarrow K\bar{\nu}) > 0.59 \times 10^{34}$ years [28]. As described, the lifetime depends on the squark and Higgsino masses μ and $\tan\beta$. In the calculation, we use $m_{\tilde{q}} = \mu = 2$ TeV, and $\tan\beta = 10$. For $\tan\beta = 20$, the lifetime along the curve can reach the current bound for the same SUSY masses. The accurate measurements of the δ_{PMNS} phase and the 23-mixing will be very important if the SUSY particles are found at the LHC.

5. Conclusions

The accurate measurement of the PMNS phase is important not only to accumulate the fundamental parameter in natural science, but also to determine the hierarchical structure of the neutrino mass matrix. If the SUSY particles are found at the LHC, the structure can open the window of the unified picture of the flavor physics. The accurate measurement of the neutrino 23-mixing angle is also important to obtain the structure. For the current experimental situation, the deviation of the maximal mixing $\theta_{23} = \pi/4$ is recently reported [29], and both $\theta_{23} > 0$ and $\theta_{23} < 0$ are possible within the statistical errors. It is expected that their accurate measurements can be done in the near future at Tokai-to-HyperKamiokande [30] and DUNE experiments [31]. The facilities can also raise the bounds of the baryon number violating nucleon decays. We learned that those two physics can be related in the $SO(10)$ model, and the accurate measurements of the oscillation parameters are important to calculate proton decay amplitudes.

The proton decay amplitudes are suppressed at a curve given in Fig. 1 if the VEV (v_R) of $\overline{126}$ Higgs representation, which reduces the rank of the $SO(10)$ gauge group, is more than about 10^{16} GeV. To realize this qualitative behavior, the size of the (1, 1) and (1, 2) elements needs to be correlated between the $\overline{126}$ coupling matrix f and the neutrino mass matrix. This algebraic reason is given in the minimal $SO(10)$ model in which the fermions couple to only $\mathbf{10} + \overline{\mathbf{126}}$ Higgs fields in Ref. [10]. We expect that this qualitative behavior can be realized even in the extended models, for example, the model that the fermions can also couple with $\mathbf{120}$ Higgs field, since it is due to the algebraic feature. The concrete quantitative numbers can be surely different due to the fit of the fermion masses and mixings. The minimal $SO(10)$ model is instructive to learn how this qualitative behavior of the suppressed proton decay is obtained. The important ingredient is $v_R \gtrsim 10^{16}$ GeV. As noted in Ref. [10], the influence of $v_R \ll 10^{16}$ GeV to low energy physics is the induction of flavor changing neutral currents in the right-handed charged sleptons, which can enlarge the electron electric dipole moment, and we can construct a big picture of the unified framework by the future experimental results.

Acknowledgements

The work of T.F. is supported in part by the Grant-in-Aid for Science Research from the Ministry of Education, Science and Culture of Japan (No. 26247036). The work of K.I. is supported in part by JSPS Research Fellowships for Young Scientists (No. 27-10232). The work of Y.M. is supported by the Excellent Research Projects of National Taiwan University under grant number NTU-ERP-105R8915.

References

- [1] P. Minkowski, Phys. Lett. B 67 (1977) 421;
T. Yanagida, Workshop on Unified Theories, KEK Report 79-18 1979, p. 95;
M. Gell-Mann, P. Ramond, R. Slansky, Supergravity, North Holland, Amsterdam, 1979, p. 315;
S.L. Glashow, 1979 Cargèse Summer Institute on Quarks and Leptons, Plenum, New York, 1980, p. 687;
R.N. Mohapatra, G. Senjanovic, Phys. Rev. Lett. 44 (1980) 912.
- [2] J. Schechter, J.W.F. Valle, Phys. Rev. D 22 (1980) 2227;
G. Lazarides, Q. Shafi, C. Wetterich, Nucl. Phys. B 181 (1981) 287;
R.N. Mohapatra, G. Senjanovic, Phys. Rev. D 23 (1981) 165.
- [3] F.P. An, et al., DAYA-BAY Collaboration, Phys. Rev. Lett. 108 (2012) 171803, arXiv:1203.1669 [hep-ex];
J.K. Ahn, et al., RENO Collaboration, Phys. Rev. Lett. 108 (2012) 191802, arXiv:1204.0626 [hep-ex];
K. Abe, et al., T2K Collaboration, Phys. Rev. Lett. 107 (2011) 041801, arXiv:1106.2822 [hep-ex].
- [4] P. Adamson, et al., MINOS Collaboration, Phys. Rev. Lett. 110 (17) (2013) 171801, arXiv:1301.4581 [hep-ex];
M. Wilking, T2K Collaboration, PoS EPS-HEP2013, 536, arXiv:1311.4114 [hep-ex], 2013;
P. Adamson, et al., NOvA Collaboration, Phys. Rev. Lett. 116 (15) (2016) 151806, arXiv:1601.05022 [hep-ex];

- P. Adamson, et al., NOvA Collaboration, Phys. Rev. D 93 (5) (2016) 051104, arXiv:1601.05037 [hep-ex].
- [5] M.C. Gonzalez-Garcia, M. Maltoni, J. Salvado, T. Schwetz, J. High Energy Phys. 1212 (2012) 123, arXiv:1209.3023 [hep-ph];
D.V. Forero, M. Tortola, J.W.F. Valle, Phys. Rev. D 90 (9) (2014) 093006, arXiv:1405.7540 [hep-ph].
- [6] F. Capozzi, G.L. Fogli, E. Lisi, A. Marrone, D. Montanino, A. Palazzo, Phys. Rev. D 89 (2014) 093018, arXiv:1312.2878 [hep-ph];
F. Capozzi, E. Lisi, A. Marrone, D. Montanino, A. Palazzo, Nucl. Phys. B 908 (2016) 218, arXiv:1601.07777 [hep-ph].
- [7] B. Dutta, Y. Mimura, R.N. Mohapatra, Phys. Rev. D 80 (2009) 095021, arXiv:0910.1043 [hep-ph];
J. High Energy Phys. 1005 (2010) 034, arXiv:0911.2242 [hep-ph];
P.S. Bhupal Dev, B. Dutta, R.N. Mohapatra, M. Severson, Phys. Rev. D 86 (2012) 035002, <http://dx.doi.org/10.1103/PhysRevD.86.035002>, arXiv:1202.4012 [hep-ph].
- [8] S. Weinberg, Phys. Rev. D 26 (1982) 287;
N. Sakai, T. Yanagida, Nucl. Phys. B 197 (1982) 533.
- [9] B. Dutta, Y. Mimura, R.N. Mohapatra, Phys. Rev. Lett. 94 (2005) 091804, arXiv:hep-ph/0412105;
B. Dutta, Y. Mimura, R.N. Mohapatra, Phys. Rev. D 72 (2005) 075009, arXiv:hep-ph/0507319.
- [10] T. Fukuyama, K. Ichikawa, Y. Mimura, Phys. Rev. D 94 (2016) 075018, arXiv:1508.07078 [hep-ph].
- [11] K.S. Babu, R.N. Mohapatra, Phys. Rev. Lett. 70 (1993) 2845, arXiv:hep-ph/9209215.
- [12] K. Matsuda, Y. Koide, T. Fukuyama, Phys. Rev. D 64 (2001) 053015, arXiv:hep-ph/0010026;
K. Matsuda, Y. Koide, T. Fukuyama, H. Nishiura, Phys. Rev. D 65 (2002) 033008, arXiv:hep-ph/0108202;
K. Matsuda, Y. Koide, T. Fukuyama, H. Nishiura, Phys. Rev. D 65 (2002) 079904 (Erratum).
- [13] T. Fukuyama, N. Okada, J. High Energy Phys. 0211 (2002) 011, arXiv:hep-ph/0205066.
- [14] T.E. Clark, T.K. Kuo, N. Nakagawa, Phys. Lett. B 115 (1982) 26;
C.S. Aulakh, R.N. Mohapatra, Phys. Rev. D 28 (1983) 217;
C.S. Aulakh, B. Bajc, A. Melfo, G. Senjanović, F. Vissani, Phys. Lett. B 588 (2004) 196, arXiv:hep-ph/0306242.
- [15] T. Fukuyama, A. Ilakovac, T. Kikuchi, S. Meljanac, N. Okada, Eur. Phys. J. C 42 (2005) 191, arXiv:hep-ph/0401213;
T. Fukuyama, A. Ilakovac, T. Kikuchi, S. Meljanac, N. Okada, Phys. Rev. D 72 (2005) 051701, arXiv:hep-ph/0412348;
B. Bajc, A. Melfo, G. Senjanovic, F. Vissani, Phys. Rev. D 70 (2004) 035007, arXiv:hep-ph/0402122;
B. Bajc, A. Melfo, G. Senjanovic, F. Vissani, Phys. Lett. B 634 (2006) 272, arXiv:hep-ph/0511352;
C.S. Aulakh, A. Girdhar, Nucl. Phys. B 711 (2005) 275, arXiv:hep-ph/0405074;
T. Fukuyama, A. Ilakovac, T. Kikuchi, S. Meljanac, N. Okada, J. Math. Phys. 46 (2005) 033505, arXiv:hep-ph/0405300.
- [16] B. Bajc, G. Senjanovic, F. Vissani, Phys. Rev. Lett. 90 (2003) 051802, arXiv:hep-ph/0210207;
B. Bajc, G. Senjanovic, F. Vissani, Phys. Rev. D 70 (2004) 093002, arXiv:hep-ph/0402140;
H.S. Goh, R.N. Mohapatra, S.-P. Ng, Phys. Lett. B 570 (2003) 215, arXiv:hep-ph/0303055;
H.S. Goh, R.N. Mohapatra, S.-P. Ng, Phys. Rev. D 68 (2003) 115008, arXiv:hep-ph/0308197;
B. Dutta, Y. Mimura, R.N. Mohapatra, Phys. Rev. D 69 (2004) 115014, arXiv:hep-ph/0402113;
B. Dutta, Y. Mimura, R.N. Mohapatra, Phys. Lett. B 603 (2004) 35, arXiv:hep-ph/0406262.
- [17] K.S. Babu, C. Macesanu, Phys. Rev. D 72 (2005) 115003, arXiv:hep-ph/0505200.
- [18] S. Bertolini, T. Schwetz, M. Malinsky, Phys. Rev. D 73 (2006) 115012, arXiv:hep-ph/0605006.
- [19] K.A. Olive, et al., Particle Data Group Collaboration, Chin. Phys. C 38 (2014) 090001.
- [20] H. Fritzsch, Z.Z. Xing, S. Zhou, J. High Energy Phys. 1109 (2011) 083, arXiv:1108.4534 [hep-ph].
- [21] C.Q. Geng, D. Huang, L.H. Tsai, Eur. Phys. J. C 75 (11) (2015) 557, arXiv:1508.02180 [hep-ph].
- [22] N. Haba, T. Horita, K. Kaneta, Y. Mimura, arXiv:1110.2252 [hep-ph].
- [23] B. Dutta, Y. Mimura, R.N. Mohapatra, Phys. Rev. D 87 (7) (2013) 075008, arXiv:1302.2574 [hep-ph].
- [24] T.P. Cheng, L.F. Li, Phys. Rev. D 22 (1980) 2860.
- [25] T. Goto, T. Nihei, Phys. Rev. D 59 (1999) 115009, arXiv:hep-ph/9808255.
- [26] H. Murayama, A. Pierce, Phys. Rev. D 65 (2002) 055009, arXiv:hep-ph/0108104.
- [27] H.S. Goh, R.N. Mohapatra, S. Nasri, S.P. Ng, Phys. Lett. B 587 (2004) 105, arXiv:hep-ph/0311330;
T. Fukuyama, A. Ilakovac, T. Kikuchi, S. Meljanac, N. Okada, J. High Energy Phys. 0409 (2004) 052, arXiv:hep-ph/0406068;
M. Severson, Phys. Rev. D 92 (9) (2015) 095026, arXiv:1506.08468 [hep-ph], arXiv:1601.06478 [hep-ph].
- [28] K. Abe, et al., Super-Kamiokande Collaboration, Phys. Rev. D 90 (7) (2014) 072005, <http://dx.doi.org/10.1103/PhysRevD.90.072005>, arXiv:1408.1195 [hep-ex].
- [29] NOvA Collaboration, Talk at Neutrino Conference in London, July 2016.
- [30] K. Abe, et al., Hyper-Kamiokande Working Group Collaboration, arXiv:1412.4673 [physics.ins-det].
- [31] R. Acciarri, et al., DUNE Collaboration, arXiv:1601.05471 [physics.ins-det].